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EINSTEIN'S THEORY OF RELATIVITY CONSIDERED FROM THE EPISTEMOLOGICAL STANDPOINT

III

THE PHILOSOPHICAL CONCEPT OF TRUTH AND THE THEORY OF RELATIVITY*

THE general principle of the relativity of knowledge received its first complete systematic working out in the history of ancient skepticism. Here it possessed, according to the fundamental character of skepticism, an exclusively negative meaning; it signified the limit in principle which is set to all knowledge and by which it is separated once for all from the definitive apprehension of the truth as "absolute." Among the skeptical "tropes" intended to show the uncertainty of sensuous and conceptual knowledge, the "trope" of $\pi\rho\acute{o}\varsigma\ \tau\iota$ stands in the first place. To know the object, our knowledge would, above all, have to be in a position to grasp it in its pure "in itself" and to separate it from all the determinations, which only belong to it relatively to us and other things. But this separation is impossible, not only actually, but in principle. For what is actually given to us only under certain definite conditions can never be made out logically as what it is in itself and under abstraction of precisely these conditions. Thus, in what we call the perception of a thing, we can never separate what belongs to the objective thing from what be-

* Translated by W. C. and M. C. Swabey.

longs to the subjective perception and contrast the two as independent factors. The form of subjective organization enters as a necessary element into all our so-called objective knowledge of things and properties. The "thing" appears, accordingly, not only differently to the various senses but it is limitlessly variable for the same organ according to the time and varying conditions of perception. For its whole character depends on the relations under which it is presented to us. No content is given us in experience unmixed with others in a purely self-identical character, but what is given us is always only a general combination of impressions. It is not one or the other, "this" or "that" definite quality, but only the reciprocal relation of the one to the other and the other to the one that is here known, indeed that is alone knowable.

Modern science has overcome the objections of skepticism to the possibility of knowledge, not by contesting their content, but by drawing from them a wholly different, indeed, opposite logical consequence. Modern science also assumes the reduction of what is taken in the naïve view of the world, as fixed and absolute "properties" of things to a system of mere relations. "With regard to the properties of the objects of the outer world," we read in, *e. g.*, Helmholtz's *Handbuch der physiologischen Optik*, "it is easy to see that all the properties we can ascribe to them, signify only the effects they produce either on our senses or on other natural objects. Color, sound, taste, smell, temperature, smoothness, solidity belong to the first class; they signify effects on our sense organs. The chemical properties are likewise related to reactions, *i. e.*, effects, which the natural body in question exerts on others. It is thus with the other physical properties of bodies, the optical, the electrical, the magnetic. Everywhere we are concerned with the mutual relations of bodies to each other, with effects which depend on the forces different bodies

exert on each other. . . . From this it follows that in fact, the properties of the objects of nature do not signify, in spite of their name, anything proper to the particular objects in and for themselves, but always a relation to a second object (including our sense organs). The type of effect must naturally always depend on the peculiarities of the effecting body as well as on those of the body on which the effect is exerted. . . . To question whether cinnabar is really red as we see it, or whether this is only an illusion of the senses, is therefore meaningless. The sensation of red is the normal reaction of normally constituted eyes to the light reflected from cinnabar. A color-blind person will see the cinnabar black or dark grey; this also is the correct reaction of his peculiarly constituted eye. . . . In itself, the one sensation is not more correct and not more false than the other. . . ." (30, p. 588f.) The old skeptical "trope," the argument of the $\pi\rho\acute{o}\varsigma\ \tau\iota$ here stands before us again in all distinctness. But renunciation of the absoluteness of things involves no longer renunciation of the objectivity of knowledge. For the truly objective element in modern knowledge of nature is not so much things as laws. Change in the elements of experience and the fact that no one of them is given in itself, but is always given with reference to something else, constitute no objection to the possibility of objectively real knowledge in so far as the laws establish precisely these relations themselves. The constancy and absoluteness of the elements is sacrificed to gain the permanency and necessity of laws. If we have gained the latter, we no longer need the former. For the objection of skepticism, that we can never know the absolute properties of things, is met by science in that it defines the concept of property in such a way that the latter involves in itself the concept of relation. Doubt is overcome by being outdone. When it is seen that "blue" can mean absolutely nothing save a relation to a seeing eye,

that "heavy" means nothing save a relation of reciprocal acceleration and that in general all "having" of properties can be resolved purely and simply into a "being-related" of the elements of experience, then the longing to grasp the ultimate absolute qualities of things, secretly at the basis of skepticism, loses its meaning. Skepticism is refuted, not by showing a way to a possible fulfillment of its demands, but by understanding and thus rendering ineffective the dogmatic import of these demands themselves.

In this transformation of the general ideal of knowledge, modern science and modern logic are both involved; the development of the one is in closest connection with that of the other. Ancient logic is entirely founded on the relation of "subject" and "predicate," on the relation of the given concept to its also given and final properties. It seeks finally to grasp the absolute and essential properties of absolute self-existent substances. Modern logic, on the contrary, in the course of its development, comes more and more to abandon this ideal and to be made into a pure doctrine of form and relation. The possibility of all determinate character of the content of knowledge is grounded, for it, in the laws of these forms, which are not reducible to mere relations of subsumption but include equally all the different possible types of relational construction and connection of elements of thought. But here doubt must begin in a new and deeper sense. If knowledge of things is understood as knowledge of laws and if an attempt is made to ground the former in the latter and to protect it from the attacks of skepticism, then what guarantees the objectivity, the truth and universality of the knowledge of laws? Do we have, in the strict sense, knowledge of laws or does all that we can gain in the most favorable case resolve itself into knowledge of particular cases? Here as we see, the problem of skepticism is reversed on the basis of the modern conception of law. What perplexed the ancient skep-

tic, who sought the substance of things, was the limitless relativity of all phenomena; it was the fact that phenomena would not remain fixed individual data, but were reduced for knowledge ever again into mere relations and relations of relations. But for the modern skeptic, to whom the objective truth, in so far as it is attainable, means the one all-inclusive and necessary law of all process, the basis of doubt lies in the fact that reality is never given us in this universal intellectual form, but is always divided and broken up into mere punctual particularities. We grasp only a here and a now, only a particularity isolated in space and time, and it is not to be seen how we could ever pass from this perception of the individual to a view of the objective form of the whole. No more than the continuum can be built up and generated by the summation of mere unextended points can a truly objective and necessary law be gained and deduced by the simple aggregation of however many particular cases. This is the form of Hume's skepticism, which is characteristically distinguished from the ancient. While the ancient skeptic could not reach the absolute substance because of the relativities in which the phenomenal world involved him, the modern skeptic fails to reach laws as universal relations because of the absolute particularities of sensation. While in the former it is the certainty of things that is questionable, in the latter it is the certainty of causal connections. The connections of processes become an illusion; what remains is only their particular atoms, the immediate data of sensation, in which all knowledge of "facts," of "matter of fact" ultimately consists.

If it is possible to overcome this essentially more radical form of skepticism also, it can only be by there being shown in it too a concealed dogmatic assumption, which lies implicitly at its basis. And this assumption consists in fact in its concept of empirical "givenness" itself. This given-

ness of "bare" impressions in which abstraction is made in principle from all elements of form and connection, proves to sharper analysis to be a fiction. When this is understood, doubt is directed, not on the possibility of knowledge, but on the possibility of the logical measuring-rod with which knowledge is measured here. Instead of the criterion of the "impression" making the universal formal relations of knowledge and its axioms questionable, the validity of this criterion must be contested on the basis of these relations. The only refuge from radical doubt lies in its being not set aside but intensified, in our learning to question, as ultimate elements of knowledge known in themselves, not only "things" and "laws" but especially sensations. The skepticism of Hume left the "simple" sensation as a completely unproblematic certainty, as a simple and unquestionable expression of "reality." While antique skepticism rested completely on the tacit assumption of absolute things, that of Hume rests on the assumption of absolute sensations. The hypostasization in the one case concerns "outer" being, in the other, "inner" being, but its general form is the same. And only by this hypostasization does the doctrine of the relativity of knowledge gain its skeptical character. Doubt does not result directly from the content of this doctrine, but, on the contrary, it depends on the fact that the doctrine is not truly and consistently thought through. As long as thought contents itself with developing, with reference to phenomena and according to demands of its own form, its logical axioms, and truth as a system of pure relations, it moves within its own circle with complete certainty. But when it affirms an absolute, whether of outer or inner experience, it is forced skeptically to annihilate itself with reference to this absolute. It strikes this absolute of things or of sensations again and again as if against the wall of the cell in which it is enclosed. Relativity, which is, fundamentally,

its immanent force, becomes its immanent limit. It is no longer the principle, which renders possible and governs the positive advance of knowledge, but is merely a necessary instrument of thought, which by that fact confesses itself not adequate to being the absolute object and the absolute truth.

This relation is indeed changed when we contrast to both the dogmatic and the skeptical concept of truth, which are united by a common root, the idealistic concept of truth. For the latter does not measure the truth of fundamental cognitions by transcendent objects, but it grounds conversely the meaning of the concept of the object on the meaning of the concept of truth. Only the idealistic concept of truth overcomes finally the conception which makes knowledge a copying, whether of absolute things or of immediately given "impressions." The "truth" of knowledge changes from a mere pictorial to a pure functional expression. In the history of modern philosophy and logic, this change is first represented in complete clarity by Leibniz, although in his case, the new thought appears in the setting of a metaphysical system, in the language of the monadological scheme of the world. Each monad is, with all its contents, a completely enclosed world, which copies or mirrors no outer being but merely includes and governs by its own law the whole of its presentations; but these different individual worlds express, nevertheless, a common universe and a common truth. This community, however, does not come about by these different pictures of the world being related to each other as copies of a common "original" but by the fact that they correspond functionally to each other in their inner relations and in the general form of their structure. For one fact, according to Leibniz, expresses another when there exists **between what can be said of the one and of the other** a constant and regular relation. Thus a perspective pro-

jection expresses its appropriate geometrical figure, an algebraic equation expresses a definite figure, a drawn model a machine; not as if there existed between them any sort of factual likeness or similarity, but in the sense that the relations of the one structure correspond to those of the other in a definite conceptual fashion. (43, VII, 263f, 44, II, 233; *cf.* 7, II, 167.) This Leibnizian concept of truth was taken up and developed by Kant who sought to free it from all the unproved metaphysical assumptions that were contained in it. In this way he gained his own interpretation of the critical concept of the object, in which the relativity of knowledge was affirmed in a far more inclusive meaning than in ancient or modern skepticism, but in which also this relativity was given a new positive meaning. The theory of relativity of modern physics can be brought without difficulty under this meaning, for, in a general epistemological regard, it is characterized by the fact that in it, more clearly and more consciously than ever before, the advance is made from the copy theory of knowledge to the functional theory. As long as physics retained the postulate of absolute space, the question still had a definite meaning as to which of the various paths of a moving body that result when we regard it from different systems of reference, represents the real and "true" motion; thus a higher objective truth had to be claimed for certain spatial and temporal values, obtained from the standpoint of certain selected systems, than for others. The theory of relativity ceases to make this exception; not that it would abandon the determinateness of natural process, but because it has at its disposal new intellectual means of satisfying this demand. The infinite multiplicity of possible systems is not identical with the infinite indeterminateness of the values to be gained in them—in so far as all these systems are to be related and connected with each other by a common rule. In this respect,

the principle of relativity of physics has scarcely more in common with "relativistic positivism," to which it has been compared, than the name. When there is seen in the former a renewal of ancient sophistical doctrines, a confirmation of the Protagorean doctrine that man is the "measure of all things," its essential achievement is mistaken.¹⁰ The physical theory of relativity teaches not that what appears to each person is true to him, but, on the contrary, it warns against taking appearances, which hold only from a particular system, as the truth in the sense of science, *i. e.*, as an expression of an inclusive and final law of experience. The latter is gained neither by the observations and measurements of a particular system nor by those of however many such systems, but only by the reciprocal coördination of the results of all possible systems. The general theory of relativity purports to show how we can gain assertions concerning all of these, how we can rise above the fragmentariness of the individual views to a total view of natural processes. (*Cf.* above.) It abandons the attempt to characterize the "object" of physics by any sort of pictorial properties, such as can be revealed in presentation, and characterizes it exclusively by the unity of the laws of nature. When, for example, it teaches that a body regarded from one system possesses spherical form and, regarded from another system, in motion relatively to the first, appears as an ellipsoid of rotation, the question can no longer be raised as to which of the two optical images here given is like the absolute form of the object, but it can and must be demanded that the multiplicity and diversity of the sensuous data here appearing can be united into a universal concept of experience. Nothing more is demanded of the critical concept of truth and the object. According to the critical view, the object is no absolute model to which our sensuous pres-

¹⁰ *Cf.* Petzoldt (61).

entations more or less correspond as copies, but it is a "concept, with reference to which presentations have synthetic unity." This concept the theory of relativity no longer represents in the form of a picture but as a physical theory, in the form of equations and systems of equations, which are co-variant with reference to arbitrary substitutions. The "relativization," which is thus accomplished, is itself of a purely logical and mathematical sort. By it the object of physics is indeed determined as the "object in the phenomenal world"; but this phenomenal world no longer possesses any subjective arbitrariness and contingency. For the ideality of the forms and conditions of knowledge, on which physics rests as a science, both assures and grounds the empirical reality of all that is established by it as a "fact" and in the name of objective validity.

IV. MATTER, ETHER AND SPACE

IN the structure of physics we must, it seems, distinguish two different classes of concepts from each other. One group of concepts concerns only the form of order as such, the other the content that enters into this form; the first determines the fundamental schema which physics uses, the other concerns the particular properties of the real by which the physical object is characterized. With regard to the pure formal concepts, they appear to persist as relatively fixed unities in spite of all changes of physical ideals in detail. In all the diversity and conflict of the systematic concepts of physics, space and time are distinguished as the ultimate, agreeing unities. They seem, in this sense, also, to constitute the real *a priori* for any physics and the presupposition of its possibility as a science. But the first step from these bare possibilities to reality, which is a matter not of the spatio-temporal form, but of the *somewhat* that is thought to be somehow "given" in space and in time, seems to force us beyond the circle of the *a priori*. Kant indeed, in the *Metaphysischen Anfangsgründen der Naturwissenschaft*, attempted an *a priori* deduction and construction of the concept of "matter" as a necessary concept of physics; but it is easy to see that this deduction does not stand on the same plane and cannot claim

the same force as the Transcendental Aesthetic or the Analytic of the Pure Understanding. He himself believed that he possessed in these deductions a philosophical grounding of the presuppositions of the science of Newton; today we recognize to an increasing extent that what he so regarded was in fact nothing but a philosophical circumlocution for precisely these presuppositions. As a fundamental definition of the physical concept of the object, the classical system of mechanics is only one structure, by the side of which there are others. Heinrich Hertz, in his new grounding of the mechanical principles, distinguished three such structures: the first is given in the Newtonian system, which is founded on the concepts of space, of time, of force, and of mass, as given presentations; the second leaves the presuppositions of space, time and mass unchanged, but substitutes for the concept of force as the mechanical "cause of acceleration" the universal concept of energy, which is divided into two different forms, potential and kinetic energy. Here, too, we have four mutually independent concepts, whose relations to each other are to constitute the content of mechanics. Hertz's own formulation of mechanics offers a third structure in which the concept of force or of energy as an independent idea is set aside and the construction of mechanics is accomplished by only three independent fundamental ideas, space, time and mass. The circle of possibilities would thus have seemed completely surveyed—had not the theory of relativity once more given a new interpretation to the mutual relation between the pure formal concepts and the physical concepts of the object and substance, and thus transformed the problem not only in content but in principle.

The concept of "nature," the gaining of which is the real methodic problem of physics, leaves room, as the history of physical thought shows, for a dualism of presuppositions, which as such seems necessary and unavoidable. Even in the first logical beginnings of genuine natural science, which are found in Greek thought, this dualism appears in full distinctness and clarity. Antique atomism, which is the first classical example of a conceptual and scientific picture of the world, can only describe and unify the "being" of nature by building it up out of two heterogeneous elements. Its view of nature is founded on the opposition of the "full" and the "void." The two, the full and the void, prove necessary elements for the constitution of the object of physics. To the being of the atom and matter as the *παμπληρες ὄν*, there is opposed by Democritus the not-being, the *μὴ ὄν* of empty space; but this being and this not-being possessed for him, however, uncontested physical truth and thus indubitable physical reality. The reality of motion was only intelligible by virtue of this dual presupposition; motion would disappear if we did not both distinguish empty space from the material filling of space and conceive the two as in inseparable mutual relation, as fundamental elements in all natural processes. At the beginning of modern times, Descartes attempted philosophically to overcome this duality in the foundations of physical thought. Proceeding from the thought of the unity of *consciousness*, he postulated also a new unity of *nature*. And this seemed to him only attainable by abandoning the opposition of the "full" and the "void," of matter and extension. The physical being of the body and the geometrical being of extension constitute one and the same object: the "sub-

stance" of a body is reduced to its spatial and geometrical determinations. Thus a new approach to physics, methodologically deeper and more fruitful, was found, the concrete realization of which, however, could not be accomplished by Descartes' physics. When Newton fought the hypothetical and speculative premises of the Cartesian physics, he also abandoned this approach. His picture of the world was rooted in the dualistic view, which was even intensified in it and which set its seal on his universal law of nature and the cosmos. On the one side, there stands space as a universal receptacle and vessel; on the other, bodies, inert and heavy masses, which enter into it and determine their reciprocal position in it on the basis of a universal dynamic law. The "quantity of matter," on the one hand, the purely spatial "distance" of the particular masses from each other, on the other, give the universal physical law of action, according to which the cosmos is constructed. Newton as a physicist always declined to ask for a further "why," for a reason for this rule. It was for him the unitary mathematical formula, which included all empirical process under it and thus perfectly satisfied the task of the exact knowledge of nature. That this formula concealed—in the expression for the cosmic *masses* and in the expression for their *distance*—two wholly different elements seemed a circumstance that no longer concerned the physicist but only the metaphysician and the speculative philosopher of nature. The proposition "*hypotheses non fingo*" cuts off any further investigation in this direction. For Newton as for Democritus, matter and space, the full and the void, form for us the ultimate but mutually irreducible elements of the physical world, the fundamental building-stones of all reality, because as equally justified and equally necessary factors, they enter into the highest law of motion taught us by experience.

If we contrast this view with the picture of the world of modern and most modern physics, there results the surprising fact that the latter seems to be again on the road to Descartes, not indeed in content, but certainly in method. It too strives from various sides toward a view in which the dualism of "space" and "matter" is cancelled, in which the two no longer occur as different classes of physical object-concepts. There now appears in the concept of the "*field*" a new mediating concept between "matter" and "empty space"; and this it is which henceforth appears with increasing definiteness as the genuine expression of the physically real since it is the perfect expression of the physical law of action. In this concept of the field, the typical manner of thought of modern physics has gained, from the epistemological standpoint, its sharpest and most distinct expression. There now takes place, starting from electrodynamics, a progressive transformation of the concept of matter. Already with Faraday, who constructed matter out of "lines of force," there is expressed the view that the field of force cannot depend on matter, but that, on the contrary, what we call matter is nothing else than specially distinguished places of this field.¹¹ In the progress of electrodynamics, this view is confirmed and assumes ever more radical expression. The doctrine is carried through more and more of a pure "field-physics," which recognizes neither bare undifferentiated space by itself nor matter by itself subsequently entering into this finished space, but which takes as a basis the intuition of a spatial manifold determined by a certain law and qualified and differentiated according to it. Thus, *e. g.*, there was established by Mie a more general form of electrodynamics on the basis of which it seemed possible to construct matter out of the field. The concept of a substance existing along with the electromagnetic field seemed unnecessary in this

¹¹ On Faraday, *cf.* Buek (4, esp. p. 41ff.); *cf.* also Weyl (83, p. 142).

approach; according to the new conception, the field no longer requires for its existence matter as its bearer, but matter is considered and treated, on the contrary, as an "outgrowth of the field." It is the last consequence of this manner of thought that is drawn by the theory of relativity. For it, too, the real difference finally disappears between an "empty" space and a space-filling substance, whether one calls this matter or ether, since it includes both moments in one and the same act of *methodic determination*. The "riddle of weight" is revealed to us, according to the fundamental thought of Einstein's theory of gravitation, in the consideration and analysis of the inner relations of measurement of the four dimensional space-time manifold. For the ten functions $g_{\mu\nu}$, which occur in the determination of the linear elements of the general theory of relativity $ds^2 = \sum_1^4 g_{\mu\nu} dx_\mu dx_\nu$ ($\mu, \nu = 1, 2, 3, 4$), represent also the ten components of the gravitation potential of Einstein's theory. It is thus the same determinations, which, on the one hand, designate and express the metrical properties of the four-dimensional space and, on the other, the physical properties of the field of gravitation. The spatio-temporal variability of the magnitudes $g_{\mu\nu}$ and the occurrence of such a field prove to be equivalent assumptions differing only in expression. Thus it is shown most distinctly that the new physical view proceeds neither from the assumption of a "space in itself," nor of "matter" nor of "force in itself"—that it no longer recognizes space, force and matter as physical objects separated from each other, but that for it exists only the unity of certain *functional relations*, which are differently designated according to the system of reference in which we express them. All dynamics tends more and more to be resolved into pure metrics, a process in which indeed the concept of metrics

undergoes, in contrast with classical geometry, an extraordinary broadening and generalization whereby the measurements of Euclidean geometry appear as only a special case within the total system of possible measurements in general. "The world," as is said by Weyl, in whose account of the general theory of relativity one can trace and survey this development most clearly," is a $(3+1)$ -dimensional metrical manifold; all physical phenomena are expressions of world metrics. . . . The dream of Descartes of a purely geometrical physics seems to be about to be fulfilled in a wonderful way, which could not have been foreseen by him." (83, p. 244; cf. p. 85ff., 170 ff.)

Just as the dualism of matter and space is superseded here by a unitary physical conception, so the opposition between "matter" and "force" is to be overcome by the principle and law of the new physics. Since Newton, as a physicist, established this opposition between the "inert masses" and the forces that affect them in the *Philosophiae Naturalis Principia Mathematica*, attempts, indeed, have not been lacking to overcome it from the philosophical and speculative side. Leibniz led the way here; but although, in his metaphysics, he wholly resolved substance into force, he retained in the construction of his mechanics, the duality of an "active" and a "passive" force, whereby matter is subsumed under the concept of the latter. The essence of matter consists in the dynamic principle immanent in it; but this expresses itself, on the one hand, in activity and striving for change, on the other hand, in the resistance which a body opposes, according to its nature, to change coming upon it from without.¹² As for Newton, the opposition in fundamental concepts, which he assumes, threatens finally to destroy the unity of the physical structure of his world; he can only retain this unity by introducing at

¹² Cf. (44), I, 204, 267ff., 332 II, 290ff., 303.

a certain place a *metaphysical* factor. The principle of the conservation of *vis viva* is disputed by him because all bodies consist of "absolutely hard" atoms, and in the rebounding of such atoms, mechanical energy must be lost; the sum total of force is in a continuous decrease, so that for its preservation the world needs from time to time a new divine impulse. (58, p. 322ff.) Kant attempted in a youthful work, the *Monadologia Physica* of the year 1756, a reconciliation and mediation between the principles of the Leibnizian philosophy and those of Newtonian mechanics; and in the *Metaphysischen Anfangsgründen der Naturwissenschaft* he returns to the attempted purely dynamic deduction and construction of matter. The "essence" of matter *i. e.*, its pure concept for experience, according to which it is nothing else than a totality of external relations, is resolved into a pure interaction of forces acting at a distance; but since these forces themselves occur in a double form, as attracting and repelling forces, the dualism is not fundamentally overcome, but is only shifted back into the concept of force itself.

Modern physics has sought, from essentially different standpoints and motives, to overcome the old opposition between matter and force, which seemed sanctioned and made eternal in the classical system of mechanics. Heinrich Hertz's *Prinzipien der Mechanik* takes the opposite course to that of previous philosophical speculation by placing the sought unity in the concept of mass, instead of in the concept of force. Along with the fundamental concepts of space and time, only the concept of mass enters into the systematic construction of mechanics. The carrying out of this view presupposes, indeed, that we do not remain with gross perceptible mass and gross perceptible motion, but supplement the sensuously given elements, which by themselves do not constitute a lawful world, by

assuming certain "concealed" masses and "concealed" motions. This supplementation takes place when it is shown to be necessary for the description and calculation of phenomena, and without arousing suspicion since mass is conceived by Hertz from the beginning merely as a definite factor of calculation. It is intended to express nothing but certain coördination of space and time values: "a particle of mass," as Hertz defines it, "is a property by which we coördinate unambiguously a certain point of space to a certain point of time (and)¹⁸ a certain point of space to every other time." (31, p. 29ff., 54.) Another attempt was made by general energetics to reach a unified foundation for physics and with it for mechanics. Inert mass appears here merely as a definite factor of energy, as the capacity-factor of the energy of motion, which with certain other capacity-factors shares with the different types of energy, *e. g.*, electricity, the empirical property of quantitative conservation. Energetics refuses to grant this law of conservation a special place and to recognize matter as a particular substance along with energy. (*Cf.* 60, p. 282ff.) But precisely in this we see very distinctly what is logically unsatisfactory, which consists in that the principle of conservation refers to wholly different moments between which an inner connection is not to be seen.

The theory of relativity brings important clarification here too in that it combines the two principles of conservation: that of the conservation of energy and that of the conservation of mass into a single principle. This result it gains by applying its characteristic manner of thought; it is led to this result by general considerations on the conditions of measurement. The demand of the theory of relativity (at first of the special theory) is that the law of the conservation of energy be valid not only with reference

¹⁸ Trans.

to any system of coördinates K but also with reference to any other in uniform rectilinear motion relatively to it; it results from this presupposition, however, combined with the fundamental equations of Maxwell's electrodynamics that when a body in motion takes up energy E_0 in the form of radiation its inert mass increases by a definite amount $\left(\frac{E}{c^2}\right)$. The mass of a body is thus a measure of its content of energy; if the energy content alters a definite amount then its mass alters proportionately.¹⁴ Its independent constancy is thus only an appearance; it holds good only in so far as the system takes up and gives off no energy. In the modern electron theory, it follows from the well-known investigation of Kaufmann that the "mass" of an electron is not unchangeable, but that it rapidly increases with the velocity of the electron as soon as the latter approaches the velocity of light. While previously a distinction had been made between a "real" and a "fictitious" mass of electrons, *i. e.*, between an inertia, which came from its ponderable mass, and another, which they possessed solely because of their motion and their electric charge, in so far as this opposed a certain resistance to every change of velocity; it now turns out that the alleged ponderable mass of the electrons is to be taken as strictly=0.

The inertia of matter thus seems completely replaced by the inertia of energy; the electron—and thus the material atom as a system of electrons—possesses no material but only "electromagnetic" mass. What was previously regarded as the truly fundamental property of matter, as its substantial kernel, is resolved into the equations of the electro-magnetic field. The theory of relativity goes further in the same direction; but it reveals in this too its peculiar *nuance* and character. This comes out especially in the process by which it gains one of its fundamental propositions: in the establishment of the equivalence of

¹⁴ Einstein (16a) and Planck (64 and 65).

phenomena of inertia and weight. Here it is at first merely a calculation, a consideration of the same phenomena from different systems of reference, which points the way. We can, as it shows, regard one and the same phenomenon now as a pure inertial movement and now as a movement under the influence of a field of gravitation according to the standpoint we choose. The equivalence of *judgment*, here indicated, grounds for Einstein the *physical identity* of phenomena of inertia and weight. If certain accelerated motions occur for an observer within his sphere of observation, he can interpret them either by ascribing them to the effects of a field of gravitation or conceive the system of reference from which he makes his measurements as in a certain acceleration. The two assumptions accomplish precisely the same in the description of the facts and can thus be applied without distinction. We can—as Einstein expresses it—produce a field of gravitation by a mere change of the system of coördinates. (17, p. 10; *cf.* 18, p. 45ff.) Hence, it follows that to attain a universal theory of gravitation we need only assume such a shift of the system of reference and establish its consequences by calculation. It suffices that in purely ideal fashion we place ourselves at another standpoint to be able to deduce certain physical consequences from this change of standpoints. What was previously done in the Newtonian theory of gravitation by the dynamics of forces is done by pure kinematics in Einstein's theory, *i. e.*, by the consideration of different systems of reference moving relatively to each other.

In emphasizing this ideal element in Einstein's theory of gravitation, the *empirical* assumption on which it rests must naturally not be forgotten. That we change in thought, by the mere introduction of a new system of reference, a field of inertia into a field of gravitation, and a field of gravitation of special structure into a field of iner-

tia, rests on the empirical equality of inert and gravitating masses of bodies, as was established with extraordinary exactitude by the investigation of Eötvös to which Einstein refers. Only the fact that gravitation imparts to all bodies found at the same place in the field of gravitation, the same amount of acceleration, and that thus it is for any definite body the same constant, *i. e.*, mass, which determines its inertial effects and its gravitational effects, renders possible that transformation of the one into the other, from which the Einstein theory starts.¹⁵ But it is especially interesting and important from a general methodological standpoint that this fundamental fact is given a completely different interpretation than in the Newtonian mechanics. What Einstein urges against the latter is that it *registered* the phenomenon of the equivalence of gravitating and inert masses, but did not interpret it. (18, p. 44.) What was established as a fact by Newton is now to be understood from principles. In this problem one can trace how gradually the question as to the "essence" of matter and of gravitation is superseded by another epistemological formulation of the question, which finds the "essence" of a physical process expressed wholly in its quantitative relations and its numerical constants. Newton never ceased to reject the question as to essence, which met him ever again, and the phrase that physics has to do merely with the "description of phenomena" was first formulated in his school and is an expression of his method.¹⁶ But so little was he able to escape this question that he expressly urged that universal attraction was not itself grounded in the essence of body, but that it came to it as something new and alien. Weight is, as he emphasizes, indeed a universal but not an essential property of matter. (59, Vol. III, p. 4.) What this distinction be-

¹⁵ For more detail, *cf.* Freundlich (24), pp. 28 and 60f. and Schlick (79), p. 27ff; *cf.* Einstein (18), p. 45ff.

¹⁶ Keill, *Introductio ad veram Physicam* (1702), (36); *cf.* 7, II, 404ff.

tween the universal and the essential means from the standpoint of the physicist, who has to do merely with the laws of phenomena, and thus with the universality of the rule to which they are subjected, is here left in the dark. Here lies a difficulty, which has been felt again and again in the tedious controversy of physicists and philosophers on the actuality and possibility of force acting at a distance. Kant, in his *Metaphysischen Anfangsgründen der Naturwissenschaft*, urges against Newton that, without the assumption that all matter merely by virtue of its essential properties exercises the action we call gravitation, the proposition that the universal attraction of bodies is proportional to their inert mass, would be a totally contingent and mysterious fact. (35, IV, p. 421.) In its solution of this problem the general theory of relativity has followed the path prescribed by the peculiarity of the physical method. The numerical *proportion*, which is universally found between inert and heavy masses becomes the expression of physical *equivalence*, of the essential likeness of the two. The theory of relativity concludes that it is the *same* quality of the body, which is expressed according to circumstances as "inertia" or as "weight." We have here in principle the same procedure before us, which, *e. g.*, in the electromagnetic theory of light led to insight into the "identity" of light waves and electrical waves. For this identity too means nothing else and nothing more mysterious than that we can represent and master the phenomena of light and the phenomena of dielectric polarization by the same *equations* and that the same numerical value results for the velocity of light and for that of dielectric polarization. This equality of values means to the physicist likeness in essence—since for him essence is defined in terms of exact determinations of measure and magnitude. In the advance to this insight, there may be traced historically a definite series of steps, a culmination of physical theories. The

physics of the eighteenth century was in general rooted in a substantialistic view. In the fundamental investigations of Sadi Carnot on thermodynamics heat was still regarded as a material, and the assumption seemed unavoidable, in understanding electricity and magnetism, of a particular electric and magnetic "matter." Since the middle of the nineteenth century, however, there appears in place of this "physics of materials," ever more definitely and distinctly the physics that has been called the "physics of principles." Here a start is not made from the hypothetical existence of certain materials and agents, but from certain universal relations, which are regarded as the criteria for the interpretation of particular phenomena. The general theory of relativity stands methodologically at the end of this series, since it collects all particular systematic principles into the unity of a supreme postulate, in the postulate not of the constancy of things, but of the invariance of certain magnitudes and laws with regard to all transformations of the system of reference.

The same evolution, that is characteristic of physical conceptual construction in general, is seen when we go from the concept of matter to the second fundamental concept of modern physics, to the concept of the ether.¹⁷ The idea of the ether, as the bearer of optical and magnetic effects was at first conceived in the greatest possible analogy and affinity with our presentations of empirically given materials and things. A sensuous description of its fundamental properties was sought by comparing it now with a perfectly incompressible fluid, now with a perfectly elastic body. But the more one attempted to work these pictures out in detail, the more distinctly was it seen that they demanded the impossible of our faculty of presentation, that

¹⁷ Here I do not go into details in the development of the hypothesis of the ether; they have been expounded from the standpoint of epistemology by *e. g.*, Aloys Müller (55, p. 90ff.) and Erich Becher (2, p. 232 ff.). On the following *cf.* *Substanzbegriff und Funktionsbegriff* (8, p. 215ff.).

they demanded the unification of absolutely conflicting properties. Thus modern physics was more and more forced to abandon in principle this sort of sensuous description and illustration. But the difficulty was unchanged also when one asked, not concerning any concrete properties of the ether, but merely concerning the abstract laws of its motion. The attempt to construct a mechanics of the ether led little by little to the sacrifice of all the fundamental principles of classical mechanics; it was seen that, really to carry it through, one would have to give up not only the principle of the equality of action and reaction, but the principle of impenetrability in which, *e. g.*, Euler saw the kernel and inclusive expression of all mechanical laws. Ether was and remained accordingly, in an expression of Planck, the "child of sorrow of the mechanical theory"; the assumption of the exact validity of the Maxwell-Hertzian differential equations for electrodynamic processes in the pure ether excludes the possibility of their mechanical explanation.¹⁸ An escape from this antinomy could only be reached by reversing the treatment. Instead of asking about the properties or constitution of the ether as a real thing, the question must be raised as to by what right here in general one seeks for a particular substance with particular material properties and a definite mechanical constitution. What if all the difficulties of the answer are based on the question itself, there being in it no clear and definite physical meaning? That is, in fact, the new position which the theory of relativity takes to the question of the ether. According to the outcome of Michelson's investigation and the principle of the constancy of the propagation of light, each observer has the right to regard his system as "motionless in the ether"; one must thus ascribe to the ether simultaneous rest with reference to wholly different sys-

¹⁸ Cf. Planck (67), p. 64ff. Lenard (45a and b), especially declares for the possibility and necessity of a "mechanics of the ether."

tems of coördinates K , K' , K'' , which are in uniform translatory motion relatively to each other. That, however, is an obvious contradiction and it forces us to abandon the thought of the ether as a somehow moving or motionless "substance," as a thing with a certain "state of motion." Physics, instead of imagining some sort of hypothetical substratum of phenomena and losing itself in consideration of the nature of this substratum, is satisfied, as it becomes a pure "physics of fields," with the body of field-equations themselves and their experimentally verifiable validity. "One cannot define," says *e. g.*, Lucien Poincaré, "ether by material properties without committing a real fallacy, and to characterize it by other properties than those, the direct and exact knowledge of which is produced for us by experiment, is an entirely useless labor condemned to sterility from the beginning. The ether is defined when we know the two fields, which can exist in it, the electric and magnetic fields, in their magnitude and direction at each point. The two fields can change; by custom we speak of a motion propagated in the ether; but the phenomenon accessible to experiment is the propagation of these changes." (75, p. 251.) Here we again face one of those triumphs of the critical and functional concept over the naïve notion of things and substances, such as are found more and more in the history of exact science. The physical rôle of the ether is played as soon as a type of exposition is found for the electrodynamic laws into which it does not enter as a condition. "The theory of relativity," remarks one of its representatives, "rests on an entirely new understanding of the propagation of electromagnetic effects in empty space; they are not carried by a medium, but neither do they take place by unmediated action at a distance. But the electromagnetic field in empty space is a thing possessing self-existent physical reality independently of all substance. Indeed, one must first accustom himself to this

idea; but perhaps this habituation will be made easier by the remark that the physical properties of this field, which are given most adequate expression in Maxwell's equations, are much more perfectly and exactly known than the properties of any substance." (Laue, 41, p. 112.) Habituation with regard to a "thing independent of any substance" can indeed be as little attributed to common human understanding as to the epistemologically trained understanding; for precisely to the latter does substance mean the *category* on the application of which rests all possibility of positing "things." But it is obvious that we have here only an inexactitude of expression and that the "independent physical reality" of the electromagnetic field can mean nothing but the reality of the relations holding within it which are expressed in the equations of Maxwell and Hertz. Since they are for us the ultimate attainable object of physical knowledge, they are set up as the ultimate attainable reality for us. The idea of the ether as an inexperiencable substance is excluded by the theory of relativity in order to give conceptual expression merely to the pure properties of empirical knowledge.

For this purpose, however, according to the theory of relativity, we do not need the fixed and rigid reference body, to which classical mechanics was ultimately referred. The general theory of relativity no longer measures with the rigid bodies of Euclidean geometry and classical mechanics, but it proceeds from a new and more inclusive standpoint in its determination of the universal linear element ds . In place of the rigid rod which is assumed to retain the same unchanging length for all times and places and under all particular conditions of measurement there now appear the curved coördinates of Gauss. If any point P of the space-time continuum is determined by the four parameters x_1, x_2, x_3, x_4 , then for it and an infinitely

close point P' there is a certain "distance" ds , which is expressed by the formula :

$$ds^2 = g_{11} dx_1^2 + g_{22} dx_2^2 + g_{33} dx_3^2 + g_{44} dx_4^2 + 2g_{12} dx_1 dx_2 + 2g_{13} dx_1 dx_3 + \dots$$

in which the magnitudes $g_{11}, g_{22}, \dots, g_{44}$ have values, which vary with the place in the continuum. In this general expression, the formula for the linear element of the Euclidean continuum is contained as a special case. We need not here go into details of this determination.¹⁹—its essential result, however, is that measurements in general differ from each other result for each place in the space-time continuum. Each point is referred, not to a rigid and fixed system of reference outside of it, but to a certain extent only to itself and to infinitely close points. Thus all measurements become infinitely fluid as compared with the rigid straight lines of Euclidean geometry, which are freely movable in space without change of form; and yet, on the other hand, all these infinitely various determinations are collected into a truly universal and unitary system. We now apply, instead of given and finite reference bodies, only "reference mollusks" as Einstein calls them; but the conceptual system of all these "mollusks" satisfies the demand for an exact description of natural processes. For the universal principle of relativity demands that all these systems can be applied as reference bodies with equal right and with the same consequences in the formulation of the universal laws of nature; the form of the law is to be completely independent of the choice of the mollusk. (18, p. 67.) Here is expressed again the characteristic procedure of the general theory of relativity; while it destroys the *thing-form* of the finite and rigid reference body it would thereby only press forward to a higher form of object, to the true *systematic form* of nature and its laws. Only by heightening and outdoing the difficulties which resulted even for classical

¹⁹ Cf. Einstein (17 and 18, pp. 59ff.); cf. below VI.

mechanics from the fact of the relativity of all motions, does it hope to find an escape in principle from these difficulties. "The clearer our concepts of space and time become," as was said in the outline of mechanics, which Maxwell has given in his short work, *Matter and Motion*, "the more do we see that everything to which our dynamic doctrines refer, belongs in a single system. At first we might think that we, as conscious beings, must have as necessary elements of our knowledge, an absolute knowledge of the place, in which we find ourselves, and of the direction in which we move. But this opinion, which was undoubtedly that of many sages of antiquity, disappears more and more from the idea of the physicist. In space, there are no milestones; one part of space is precisely like any other part, so that we cannot know where we are. We find ourselves in a waveless sea without stars, without compass and sun, without wind and tide, and cannot say in what direction we move. We have no log that we can cast out to make a calculation; we can indeed determine the degree of our motion in comparison with neighboring bodies, but we do not know what the motion in space of these bodies is." (51, p. 92f.). From this mood of "*ignorabimus*," into which physics was sinking more and more, only a theory could free it which grasped the problem at its root; and, instead of modifying the previous solutions, transformed fundamentally the formulation of the question. The question of absolute space and absolute motion could receive only the solution which had been given to the problem of the perpetual *mobile* and the squaring of the circle. It had to be made over from a mere negative expression into a positive expression, to be changed from a limitation of physical knowledge to a principle of such knowledge, if the true philosophic import, which was concealed in it, was to be revealed.

V

THE CONCEPTS OF SPACE AND TIME OF CRITICAL IDEALISM
AND THE THEORY OF RELATIVITY

WE have hitherto sought primarily to understand the special and general theory of relativity on its physical side. In fact, this is the standpoint from which it must be judged and one does it poor service if one seeks precipitately to interpret its results in purely "philosophical" or indeed in speculative and metaphysical terms. The theory contains not one concept, which is not deducible from the intellectual means of mathematics and physics and perfectly representable in them. It only seeks to gain full consciousness of precisely these intellectual means by seeking not only to represent the result of physical measurement, but to gain fundamental clarity concerning the form of any physical measurement and its conditions.

Thereby it seems indeed to come into the immediate neighborhood of the critical and transcendental theory, which is directed on the "possibility of experience"; but it is nevertheless different from it in its general tendency. For, in the language of this transcendental criticism, the doctrine of space and time developed by the theory of relativity is a doctrine of empirical space and empirical time, not of pure space and pure time. As far as concerns this point, there is scarcely possible a difference of opinion; and, in fact, all critics, who have compared the Kantian and the Einstein-Minkowski theories of space and time seem to have reached essentially the same result.²⁰ From the

²⁰ Cf. esp. Natorp, (56, p. 392ff.). Höningwald (33, p. 88ff.). Frischeisen-Köhler (26, p. 323ff.) and more recently Sellien (81, p. 14ff.).

standpoint of a strict empiricism, one could attempt to dispute the possibility of a doctrine of "pure space" and of "pure time"; but the conclusion cannot be avoided that in so far as such a doctrine is justified, it must be independent of all results of concrete measurement and of the particular conditions, which prevail in the latter. If the concepts of pure space and pure time have in general any definite justified meaning, to use a phrase of the theory of relativity, then this meaning must be invariant with regard to all transformations of the doctrine of the empirical measurement of space and time. The only thing that such transformations can and will accomplish is that they teach us to draw the line more sharply between what belongs to the purely philosophical, "transcendental," criticism of the concepts of space and time and what belongs merely to the particular applications of these concepts. Here, in fact, the theory of relativity can perform an important indirect service for the general criticism of knowledge,—if we resist the temptation to translate its propositions directly into propositions of the criticism of knowledge.

Kant's doctrine of space and time developed to a large extent on the basis of physical problems, and the conflict carried on in the natural science of the eighteenth century on the existence of absolute time and absolute space affected him keenly from the beginning. Before he approached the problems of space and time as a critical philosopher, he had himself lived through the various and opposite solutions by which contemporary physics sought to master these problems. Here, at first, contrary to the dominant scholastic opinion, he took his stand throughout on the basis of the relativistic view. In his *Neuen Lehrbegriff der Bewegung und der Ruhe* of the year 1758, the thirty-four year old Kant set up the principle of the relativity of all motion with all decisiveness and from it at-

tacked the traditional formulation of the principle of inertia. "Now I begin to see," he says after he has illustrated the difficulties of the concept of "absolute motion" with well-known examples, "that I lack something in the expression of motion and rest. I should never say: a body rests without adding with regard to what thing it rests, and never say that it moves without at the same time naming the objects with regard to which it changes its relation. If I wish to imagine also a mathematical space free from all creatures as a receptacle of bodies, this would still not help me. For by what should I distinguish the parts of the same and the different places, which are occupied by nothing corporeal?" (35, II, 19.) But Kant, in his further development did not at first remain true to the norm, which he here set up so decisively and of which a modern physicist has said that it deserves to be set up in iron letters over each physical lecture hall.²¹ He ventured to abandon the concept of inertial force, of *vis inertiae*; he refused to pour his thoughts on the principles of mechanics "into the mill of the Wolffian or of any other famous system of doctrine." But while he opposed in this way the authority of the leading philosophers, he could not permanently withdraw himself from the authority of the great mathematical physicists of his time. In his *Versuch, den Begriff der negativen Grössen in die Weltweisheit einzuführen* of the year 1763, he took his place at the side of Euler to defend with him the validity of the Newtonian concepts of absolute space and absolute time, and six years later, in his essay on the first grounds of the difference of regions in space (1769), he sought to support the proof, that Euler had attempted, of the existence of absolute space from the principles of mechanics, by another, purely geometrical consideration, which "would give practical geometers a conclusive reason to be able to affirm the reality of their

²¹ Streintz (82), p. 42.

absolute space with the "evidence" which is customary to them." (35, II, 394.) But this is indeed only an episode in Kant's evolution; for only a year later the decisive critical turn in the question of space and time had taken place in his Inaugural Dissertation of the year 1770. By it the problem receives an entirely new form; it is removed from the field of physics to that of "transcendental philosophy" and must be considered and solved according to the general principles of the latter.

But the transcendental philosophy does not have to do primarily with the reality of space or of time, whether these are taken in a metaphysical or in a physical sense, but it investigates the objective *significance* of the two concepts in the total structure of our empirical knowledge. It no longer regards space and time as things, but as "sources of knowledge." It sees in them no independent objects, which are somehow present and which we can master by experiment and observation, but "conditions of the possibility of experience," conditions of experiment and observation themselves, which again for their part are not to be viewed as things.

What—like time and space—makes possible the positing of objects can itself never be given to us as a particular object in distinction from others. For the "*forms*" of possible experience, the forms of intuition as well as the pure concepts of the understanding, are not met again as *contents* of real experience. Rather the only possible manner in which we can ascribe any sort of "objectivity" to these forms must consist in that they lead to certain *judgments* to which we must ascribe the values of necessity and universality. The meaning is thus indicated, in which one can henceforth inquire as to the objectivity of space or time. Whoever demands absolute thing-like correlates for them strains after shadows. For their whole "being" consists in the meaning and function they possess for the

complexes of judgments, which we call science, whether geometry or arithmetic, mathematical or empirical physics. What they can accomplish as presuppositions in this connection can be exactly determined by transcendental criticism; what they are as things in themselves is a vain and fundamentally unintelligible question. This basic view comes out clearly even in the Inaugural Dissertation. Even here absolute space and time possessing an *existence* separate from empirical bodies and from empirical events, are rejected as nonentities, as mere conceptual fictions (*inane rationis commentum*.) The two, space and time, signify only a fixed law of the mind, a schema of connection by which what is sensuously perceived is set in certain relations of coexistence and sequence. Thus the two have, in spite of their "transcendental ideality," "empirical reality," but this means always only their validity for all experience, which however must never be confused with their existence as isolated objective contents of this experience itself. "Space is merely the form of external intuition (formal intuition) and not a real object that can be perceived by external intuition. Space, as prior to all things which determine it (fill or limit it), or rather which give an empirical intuition determined by its form, is, under the name of absolute space, nothing but a mere possibility of external phenomena. . . . If we try to separate one from the other, and to place space outside all phenomena, we arrive at a number of empty determinations of external intuition, which, however, can never be possible perceptions; for instance, motion or rest of the world in an infinite empty space, *i. e.*, a determination of the mutual relation of the two, which can never be perceived, and is therefore nothing but the predicate of a mere idea." (34, p. 457; Müller trans., p. 347.)

Accordingly, when Einstein characterizes as a fundamental feature of the theory of relativity that it takes from

space and time "the last remainder of *physical objectivity*," it is clear that the theory only accomplishes the most definite application and carrying through of the standpoint of critical idealism within empirical science itself. Space and time in the critical doctrine are indeed distinguished in their validity as types of order from the contents, which are ordered in them; but these forms possess for Kant a separate existence neither in the subjective nor in the objective sense. The conception, that space and time as subjective forms into which sensations enter "lie ready in the mind" before all experience, not as "physical" but as "psychical" realities, today scarcely needs refutation. This conception indeed seems to be indestructible, although Fichte poured upon it his severe but appropriate scorn; but it disappears of itself for everyone who has made clear to himself even the first conditions of the transcendental formulation of the question in opposition to the psychological. The meaning of the principle of order can in general be comprehended only in and with what is ordered; in particular, it is urged in the case of the measurement of time that the determination of the temporal positions of particular empirical objects and processes cannot be derived from the relations of the phenomena to absolute time, but that conversely the phenomena must determine and make necessary their positions in time for each other. "This unity in the determination of time is dynamical only, that is, time is not looked upon as that in which experience assigns immediately its place to every existence, for this would be impossible; because absolute time is no object of perception by which phenomena could be held together; but the rule of the understanding through which alone the existence of phenomena can receive synthetical unity in time determines the place of each of them in time, therefore *a priori* and as valid for all time." (34, p. 245 and 262; cf. 56, p. 332; cf. Müller trans., p. 175.)

It is such a "rule of the understanding," in which is expressed the synthetic unity of phenomena and their reciprocal dynamical relation, on which rests all empirical spatial order, all objective relations of spatial "community" in the corporeal world. The "*communio spatii*," *i. e.*, that *a priori* form of coexistence, which in Kant's language is characterized as "pure intuition" is, as he expressly urges, only empirically knowable for us by the *commercium* of substances in space, *i. e.*, by a whole of physical effects, that can be pointed out in experience. We read, in a passage of the *Critique of Pure Reason*, which appears especially significant and weighty in connection with the development of the modern theory of relativity: "The word *communion* (*Gemeinschaft*), may be used in two senses, meaning either *communio* or *commercium*. We use it here in the latter sense: as a dynamical communion, without which even the local *communio spatii* could never be known empirically. We can easily perceive in our experience, *that continuous influences only can lead our senses in all parts of space from one object to another; that the light which plays between our eyes and celestial bodies produces a mediate communion between us and them*, and proves the coexistence of the latter; that we cannot change any place empirically (perceive such a change) unless matter itself renders the perception of our own place possible to us, and that by means of its reciprocal influence only matter can evince its simultaneous existence, and thus (though mediately only) its coexistence, even to the most distant objects." (34, p. 260; *cf.*, Müller trans., p. 173f.) The spatial order of the corporeal world, in other words, is never given to us directly and sensuously, but is the result of an intellectual construction, which takes its start from certain empirical laws of phenomena and from that point seeks to advance to increasingly general laws, in which finally is grounded

what we call the unity of experience as a spatio-temporal unity.

But is there not found in this last expression the characteristic and decisive opposition between the theory of space and time of critical idealism and the theory of relativity? Is not the essential result of this theory precisely the destruction of the unity of space and time demanded by Kant? If all measurement of time is dependent on the state of motion of the system from which it is made there seem to result only infinitely many and infinitely diverse "place-times," which, however, never combine into the unity of "the" time. We have already seen, however, that this view is erroneous, that the destruction of the substantialistic unity of space and time does not destroy their functional unity but rather truly grounds and confirms it. (*Cf.* above, p. 33ff., p. 54ff.) In fact, this state of affairs is not only granted by the representatives of the theory of relativity among the physicists, but is expressly emphasized by them. "The boldness and the high philosophical significance of Einstein's doctrine consists," we read, *e. g.*, in the work of Laue, "in that it clears away the traditional prejudice of one time valid for all systems. Great as the change is, which it forces upon our whole thought, there is found in it not the slightest epistemological difficulty. For in Kant's manner of expression time is, like space, a pure form of our intuition; a schema in which we must arrange events, so that in opposition to subjective and highly contingent perceptions they may gain objective meaning. This arranging can only take place on the basis of empirical knowledge of natural laws. The place and time of the observed change of a heavenly body can only be established on the basis of optical laws. That two differently moving observers, each one regarding himself at rest, should make this arrangement differently on the basis of the same laws of nature, contains no logical impossibil-

ity. Both arrangements have, nevertheless, objective meaning since there may be deduced exactly from each of them by the derivative transformation formulae that arrangement valid for the other moving observer." (40, p. 36f.) This one-to-one correlation and not the oneness of the values gained in the different systems, is what remains of the notion of the "unity of time"; but precisely in it is expressed all the more sharply the fundamental view that this unity is not to be represented in the form of a particular objective content, but exclusively in the form of a system of valid relations. The "dynamic unity of temporal determinations" is retained as a postulate; but it is seen that we cannot satisfy this postulate if we hold to the laws of the Newtonian mechanics, but that we are necessarily driven to a new and more universal and concrete form of physics. The "objective" determination shows itself thus to be essentially more complex than the classical mechanics assumed, which believed it could literally grasp with its hands the objective determination in its privileged systems of reference. That a step is thereby taken beyond Kant is incontestible; for he shaped his "Analogies of Experience" essentially on the three fundamental Newtonian laws: the law of inertia, the law of the proportionality of force and acceleration, and the law of the equality of action and reaction. But in this very advance the doctrine that it is the "rule of the understanding," that forms the pattern of all our temporal and spatial determinations, is verified anew. In the special theory of relativity, the principle of the constancy of the velocity of light serves as such a rule; in the general theory of relativity this principle is replaced by the more inclusive doctrine that all Gaussian coördinate systems are of equal value for the formulation of the universal natural laws. It is obvious that we are not concerned here with the expression of an empirically observed fact, but with a principle which the understanding

uses hypothetically as a norm of investigation in the interpretation of experience, for how could an infinite totality be "observed"? And the meaning and justification of this norm rest precisely on the fact that only by its application could we hope to regain the lost unity of the object, namely, the "synthetic unity of phenomena according to temporal relations." The physicist now depends neither on the constancy of those objects with which the naïve sensuous view of the world rests nor on the constancy of particular spatial and temporal measurements gained from a particular system, but he affirms, as a condition of his science, the existence of "universal constants" and universal laws, which retain the same values for all systems of measurement.

In his *Metaphysischen Anfangsgründen der Naturwissenschaft*, Kant, returning to the problem of absolute space and time, formulates a happy terminological distinction, which is suited to characterize more sharply the relation of critical idealism to the theory of relativity. Absolute space, he urges here too, is *in itself nothing and indeed no object*; it signifies only a *space relative to every other* which I can think outside of any given space. To make it a real thing means to confuse the *logical universality* of any space with which I can compare any empirical space as included in it with the *physical universality* of real extension and to misunderstand the Idea of reason. The true logical universality of the Idea of space thus not only does not include its physical universality, as an all inclusive container of things, but it is precisely of a sort to exclude it. We should, in fact, *conceive* an absolute space, *i. e.*, an ultimate unity of all spatial determinations; but not in order to know the absolute movements of empirical bodies, but to represent in the same "all movements of the material as merely relative to each other, as alternatively reciprocal, but not as absolute motion or rest." "Absolute space is thus neces-

sary not as a concept of a real object, but as an Idea, which should serve as a rule for considering all motions in it as merely relative, and all motion and rest must be reduced to the absolute space, if the phenomena of the same are to be made into a definite concept of experience that unifies phenomena." (35, IV, 383f., 472f.) The logical universality of such an idea does not conflict with the theory of relativity; it starts by regarding all motions in space as merely relative because only in this way can it combine them into a definite concept of experience, that unifies all phenomena. On the basis of the demand for the totality of determinations it negates every attempt to make a definite particular system of reference the norm for all the others. The one valid norm is merely the idea of the unity of nature, of exact determination itself. The mechanical view of the world is overcome from this standpoint. The "unity of nature" is grounded by the general theory of relativity in a new sense, since it includes under a supreme principle of knowledge along with the phenomena of gravitation, which form the real classical field of the older mechanics, the electrodynamic phenomena. That in order to advance to this "logical universality of the Idea," many trusted presentational pictures must be sacrificed need not disturb us; this can affect the "pure intuition" of Kant only in so far as it is misunderstood as a mere picture and not conceived and estimated as a constructive method.

In fact, the point at which the general theory of relativity must implicitly recognize the methodic presupposition, which Kant calls "pure intuition" can be pointed out exactly. It lies, in fact, in the concept of "coincidence" to which the general theory of relativity ultimately reduces the content and the form of all laws of nature. If we characterize events by their space-time coördinates $x_1, x_2, x_3, x_4, x'_1, x'_2, x'_3, x'_4, \text{etc.}$, then, as it emphasizes, everything that physics can teach us of the "essence" of natural

processes consists merely in assertions concerning the coincidences or meetings of such points. We reach the construction of physical time and of physical space merely in this way; for the whole of the space-time manifold is nothing else than the whole of such coördinations.²² Here is the point at which the ways of the physicist and of the philosopher definitely part, without their being thereby forced into conflict. What the physicist calls "space" and "time" is for him a concrete measurable manifold, which he gains as the *result* of coördination, according to law, of the particular points; for the philosopher, on the contrary, space and time signify nothing else than the forms and *modi*, and thus the presuppositions, of this coördination itself. They do not result for him from the coördination, but they *are* precisely this coördination and its fundamental directions. It is coördination from the standpoint of coexistence and adjacency or from the standpoint of succession, which he understands by space and time as "forms of intuition." In this sense, both are expressly defined in the Kantian Inaugural Dissertation. "*Tempus non est objectivum aliquid et reale . . . sed subjectiva conditio, per naturam mentis humanæ necessaria, quaelibet sensibilia certa lege sibi coordinandi et intuitus purus . . . Spatium est . . . subjectivum et ideale et e natura mentis stabili lege proficiscens veluti schema omnia omnino externe sensa sibi coordinandi.*" (35; II, 416, 420.) Whoever recognizes this law and this schema, this possibility of relating point to point and connecting them with each other, has recognized space and time in their "transcendental meaning," for we can abstract here from any psychological by-meaning of the concept of form of intuition. We can thus conceive the "world-points" x_1 x_2 x_3 x_4 and the world-lines, which result from them, so abstractly that we understand under the values x_1 x_2 x_3 x_4 nothing but certain mathemati-

²² Einstein (17), p. 13f.; (18), p. 64.

cal parameters; the "meeting" of such world-points involves a comprehensible meaning only if we take as a basis that "possibility of succession," which we call time. A coincidence, which is not to mean identity, a unification, which is still a separation, since the same point is conceived as belonging to different lines: all this finally demands that synthesis of the manifold, for which the term "pure intuition" was formulated. The most general meaning of this term, which indeed was not always grasped by Kant with equal sharpness, since more special meanings and applications were substituted involuntarily in his case, is merely that of the serial form of coexistence and of succession. Nothing is thereby presupposed concerning special relations of measurement in the two, and in so far as these depend in particular on the relations of the physical in space, we must guard against seeking to find an exhaustive determination in the mere "forms of possibility" of the relations of the "real." (*Cf.* below VI.) When, *e. g.*, in the mathematical foundations of the theory of relativity the formula is deduced for the "distance" of the two infinitely close points $x_1 \ x_2 \ x_3 \ x_4$, and $x_1+dx_1, x_2+dx_2, x_3+dx_3, x_4+dx_4$, this cannot indeed be conceived as a rigid Euclidean distance in the ordinary sense, since there is involved in it, by the addition of time as a fourth dimension, not a magnitude of space but rather one of motion; but the fundamental form of coexistence and succession and their reciprocal relation and "union" is unmistakably contained in this expression of the general linear element. Not that the theory, as has been occasionally objected, presupposes space and time as something already given, for it must be declared free of this epistemological circle, but in the sense that it cannot lack the form and function of spatiality and temporality in general.

What seems to render understanding difficult at this point between the physicist and the philosopher is the fact

that a common problem is found here, which both approach from entirely different sides. The process of measurement interests the critic of knowledge only in so far as he seeks to survey in systematic completeness the concepts, which are used in this process, and to define them in the utmost sharpness. But any such definition is unsatisfying and fundamentally unfruitful to the physicist as long as it is not connected with any definite indication as to how the measurement is to be made in the concrete particular case. "The concept exists for the physicist," says Einstein in one place neatly and characteristically, "only when the possibility is given of finding out in the concrete case whether the concept applies or not." (18, p. 14.) Thus the concept of simultaneity, for example, only receives a definite meaning, when a method is given by which the temporal coincidence of two events is determined by certain measurements, by the application of optical signals; and the difference which is found in the results of this measurement seems to have as a consequence the ambiguity of the concept. The philosopher has to recognize unconditionally this longing of the physicist for concrete determinateness of concepts; but he is ever again brought to the fact that there are ultimate ideal determinations without which the concrete cannot be conceived and made intelligible. To make clear the opposition in formulation of the question which is here fundamental, one can contrast to Einstein's expression one of Leibniz. "*On peut dire,*" we read in Leibniz' *Nouveaux Essais*, "*qu'il ne faut point s'imaginer deux étendues, l'une abstraite, de l'espace, l'autre concrète, du corps; le concret n'étant tel que par l'abstrait.*" (43, V, 115.) As we see, it is the unity of the abstract and the concrete, of the ideal and the empirical in which the demands of the physicist and the philosopher agree; but while the one goes from experience to the idea, the other goes from the idea to experience. The theory of

relativity holds fast to the "pre-established harmony between pure mathematics and physics"; Minkowski, in the well-known concluding words of his lecture, "Space and Time," has expressly taken up again and brought to honor this Leibnizian term. But this harmony is for the physicist the incontestable premise from which he strives to reach the particular consequences and applications, while for the critic of knowledge the "possibility" of this harmony constitutes the real problem. The basis of this possibility he finds ultimately in the fact that any physical assertion, even the simplest determination of magnitude established by experiment and concrete measurement, is connected with universal conditions, which gain separate treatment in pure mathematics, that any physical assertion involves certain logico-mathematical constants. If we desire to bring all of these constants into a short formula, we can point out the concept of number, the concept of space, the concept of time, and the concept of function as the fundamental elements, which enter as presuppositions into every question which physics can raise. None of these concepts can be spared or be reduced to another so that, from the standpoint of the critique of cognition, each represents a specific and characteristic motive of thought; but, on the other hand, each of them possesses an actual empirical use only along with the others and in systematic connection with them. The theory of relativity shows with especial distinctness how, in particular, the thought of function is effective as a necessary motive in each spatio-temporal determination. Thus physics knows its fundamental concepts never as logical "things in themselves," but only in their reciprocal combination; it must, however, be open to epistemology to analyze this product into its particular factors. It thus cannot admit the proposition that the meaning of a concept is identical with its concrete application, but it will con-

versely insist that this meaning must be already established before any application can be made. Accordingly, the thought of space and time in their meaning as connecting forms of order is not first created by measurement but is only more closely defined and given a definite content. We must have grasped the concept of the "event" as something spatio-temporal, we must have understood the meaning expressed in it, before we can ask as to the coincidence of events and seek to establish it by special methods of measurement.

In general, physics sees itself placed by its fundamental problem from the beginning between two realms, which it has to recognize and between which it has to mediate without asking further as to their "origin." On the one side, stands the manifold of data of sensation, on the other a manifold of pure functions of form and order. Physics, as an empirical science, is equally bound to the "material" content, which sense perception offers it, and to these formal principles in which is expressed the universal conditions of the "possibility of experience." It has to "invent" or to derive deductively the one as little as the other, *i. e.*, neither the whole of empirical contents nor the whole of characteristic scientific forms of thought, but its task consists in progressively relating the realm of "forms" to the data of empirical observation and, conversely, the latter to the former. In this way, the sensuous manifold increasingly loses its "contingent" anthropomorphic character and assumes the imprint of thought, the imprint of systematic unity of form. Indeed "form," just because it represents the active and shaping, the genuinely creative element, must not be conceived as rigid, but as living and moving. Thought comprehends more and more that form in its peculiar character cannot be given to it at one stroke, but that the existence of form is only revealed to it in the becoming of form and in the law of this becoming. In

this way, the history of physics represents not a history of the discovery of a simple series of "facts," but the discovery of ever new and more special means of thought. But in all change of these means of thought there is nevertheless revealed, as surely as physics follows the "sure course of a science," the unity of those methodic principles upon which rests the formulation of its question. In the system of these principles, space and time take their fixed place, although they are not to be conceived as fixed things or contents of presentation. The ancient view believed that it possessed and encompassed the spatio-temporal unity of being directly in presentation. To Parmenides and fundamentally the whole ancient world being was given "like the mass of a well-rounded sphere." With the reform of Copernicus, the security of this possession was gone once for all. Modern science knows that there is a definite spatio-temporal order of phenomena for knowledge only in so far as knowledge progressively establishes it, and that the only means of establishing it consists in the scientific concept of law. But the problem of such a general orientation remains for thought and becomes the more urgent the more thought knows it as a problem never to be solved definitively. Precisely because the unity of space and time of empirical knowledge seems to flee eternally before all our empirical measurements, thought comprehends that it must seek it eternally and that it must avail itself of new and ever sharper instruments. It is the merit of the theory of relativity not only to have proved this in a new way but also to have established a principle, *i. e.*, the principle of the co-variancy of the universal laws of nature with regard to all arbitrary substitutions, by which thought can master, out of itself, the relativity which it calls forth.

In the analysis of spatial and temporal measurements, made by the theory of relativity this fundamental relation can be traced in detail. This analysis does not begin by

accepting the concept of the "simultaneity" of two processes as a self-evident and immediately known datum, but by demanding an explanation of it—an explanation, which, as a physical explanation, cannot consist in a general conceptual definition, but only in the indication of the concrete methods of measurement, by which "simultaneity" can be empirically pointed out. The simultaneity of such processes as take place practically in "the same" point of space or in immediate spatial adjacency is at first presupposed; we assume, as Einstein explains, the determinability of "simultaneity" for events, which are immediately adjacent spatially, or, more exactly, for events in immediate spatio-temporal adjacency (coincidence), without defining this concept. (17, § 3.) In fact, recourse here to a mediating physical method of measurement seems neither desirable nor possible; for any such method would always presuppose the possibility of making a temporal coördination between diverse events, thus, *e. g.*, of establishing "the simultaneity" of a definite event with a certain position of the hands of a clock found at the "same" place. The real problem of the theory of relativity begins only when we are no longer concerned with temporally connecting spatially adjacent series of events with each other, but rather series of events spatially remote from each other. If we assume that there is established for the two points of space A and B a certain "place-time," then we possess only an "A-time" and "B-time" but no time *common* to A and B. And it is seen that every attempt to establish such a common time as an empirically measurable time, is bound to a definite empirical presupposition concerning the velocity of light. The assumption of the uniform velocity of light enters implicitly into all our assertions concerning the simultaneity of what is spatially distant. A time common to A and B is gained when one establishes *by definition* that the "time," which light takes in going

from A to B is equal to the "time," which it takes in going from B to A. Let us assume that a ray of light is sent at A-time t_A from a clock found in A to B, and then at B-time, t_B , the ray of light is reflected to A and reaches A again at A-time, t'_A ; then we establish by definition that the two clocks of A and B are to be called "synchronous" if $t_B - t = t'_A - t_B$. Thus for the first time an exact determination is made of what we are to understand by the "time" of an event and by the "simultaneity" of two processes; "the time" of an event is what is told us by a motionless clock found at the place of the event simultaneously with the event, a clock which runs synchronously with a certain motionless clock and indeed synchronously with the latter at all times." (16, p. 28f.)

That the "forms" of space and time as definite forms of the coördination of different contents already enter into the concrete determinations, which are here made for the procedure of the physical measurement of time, scarcely needs special explanation. The two are immediately assumed in the concept of the "place-time"; for the possibility is involved in it of grasping a definitely distinguished "now" in a definitely distinguished "here." This "here" and "now" does not signify indeed the whole of space and time, to say nothing of all the concrete relations within the two to be established by measurement; but it represents the first foundation, the unavoidable basis of the two. The first primitive difference, which is expressed in the mere positing of a "here" and a "now" remains thus, for the theory of relativity, too, an indefinable on which it grounds its complex physical definitions of space and time values. And while for these definitions it appeals to a definite assumption concerning the law of the propagation of light, this, too, involves the presupposition that a certain condition that we call "light" occurs *in succession* at different *places* and according to a definite rule, in which what space and

time mean as mere schemata of coördination, is obviously contained. The epistemological problem seems indeed to be heightened when we reflect on the reciprocal relation of space and time values in the fundamental equations of physics. What is given in these equations is the four-dimensional "world," the continuum of events in general, the temporal determinations in this continuum not being separated from the spatial. The intuitive difference between a spatial distance and a temporal duration, which we believe ourselves to grasp immediately, plays no rôle in this purely mathematical determination. According to the temporal equation of the Lorentz-transformation:

$$t' = \frac{t - \frac{v}{c^2} x}{\sqrt{1 - \frac{v^2}{c^2}}}$$

the time differential $\Delta t'$ between two events with reference to K' does not disappear when the time differential Δt of the same disappears with reference to K ; the purely spatial distance of two events with reference to K has as a consequence in general the temporal sequence of the same with reference to K' . This leveling of space and time values is developed even further in the general theory of relativity. Here it is seen to be impossible to construct a reference system out of fixed bodies and clocks of such a sort that place and time are directly indicated by a fixed arrangement of measuring rods and clocks relatively to each other; but each point of the continuous series of events is correlated with four numbers, x_1, x_2, x_3, x_4 , which possesses no direct physical meaning, but only serve to enumerate the points of the continuum in a definite but arbitrary way. This correlation need not have such properties that a certain group of values x_1, x_2, x_3 must be understood as the spatial coördinates and opposed to the "temporal" coördinate x_4 . (18, p. 38, 64.) The demand of Minkowski that "space for itself and time for itself be completely degraded

to shadows" and that only "a sort of union of the two shall retain independence" seems thus now to be realized in all strictness. Now at any rate, this demand contains nothing terrible for the critical idealist, who has ceased to conceive space and time as things in themselves or as given empirical objects. For the realm of ideas is for him a "realm of shadows," as Schiller called it, since no pure idea corresponds directly to a concrete real object, but rather the ideas can always only be pointed out in their systematic community, as fundamental moments of concrete objective knowledge. If it thus appears that physical space and time measurements can be assumed only as taking place in common, the difference in the fundamental character of space and time, of order in coexistence and succession is not thereby destroyed. Even if it is true that, as Minkowski urges, no one has *perceived* a place save at a time and a time save at a place, there remains a difference between what is to be *understood* by spatial and by temporal discrimination. The factual interpenetration of space and time in all empirical physical measurements does not prevent the two from being different in principle, not as objects, but as types of objective discrimination. Although two observers in different systems K and K' can assume the arrangement of the series of events in the orders of space and time to be different, it is still always a *series of events* and thus a continuum both spatial and temporal, which they construct in their measurements. Each observer distinguishes from his standpoint of measurement a continuum, which he calls "space," from another, which he calls "time"; but he can, as the theory of relativity shows, not assume without further consideration that the arrangement of phenomena in these two schemata must be similar from each system of reference. There may thus, according to Minkowski's "world postulate," be given only the four-dimensional word in space and time, and "the pro-

jection into space and time" may be possible "with a certain freedom"; this only affects the different spatio-temporal interpretations of phenomena, while the difference of the form of space from that of time is unaffected.

For the rest, here too the transformation-equation re-establishes objectivity and unity, since it permits us to translate again the results found in one system into those of the other. Also, if one seeks to clarify the proposition of Minkowski that only the inseparable union of space and time possesses independence, by saying that this union itself, according to the results of the general theory of relativity, becomes a shadow and an abstraction, and that only the unity of space, time and things possesses independent reality,²³ then this classification only leads us back again to our first epistemological insight. For that neither "pure space" nor "pure time" nor the reciprocal connection of the two, but only their realization in some empirical material gives what we call "reality," *i. e.*, the physical being of things and of events, belongs to the fundamental doctrines of critical idealism. Kant himself did not weary of referring repeatedly to this indissoluble connection, this reciprocal correlation of the spatio-temporal form and the empirical content in the existence and structure of the *world of experience*. "To give an object," we read, "if this is not meant again as mediate only, but if it means to represent something immediately in intuition, is nothing else but to refer the representation of the object to experience. . . . Even space and time, however, pure these concepts may be of all that is empirical, and however certain it is that they are represented in the mind entirely *a priori*, would lack nevertheless all objective validity, all sense and meaning, if we could not show the necessity of their use with reference to all objects of experience. Nay, their representation is a pure schema, always referring to

²³ See Schlick (79), p. 51; *cf.* p. 22.

that reproductive imagination, which calls up the objects of experience, without which objects would be meaningless." (34, p. 195; *cf.* Müller trans., p. 127f.) The "*ideal*" meaning, that space and time possess "in the mind" thus does not involve any sort of *particular* existence, which they would possess prior to things and independently of them, but it rather expressly denies it—the ideal separation of pure space and pure time from things (more exactly, from empirical phenomena), not only permits but demands precisely their empirical "union." This union the general theory of relativity has verified and proved in a new way, since it recognizes more deeply than all preceding physical theories the dependency belonging to all empirical measurement, to all determination of concrete spatio-temporal relations.²⁴ The relation of experience and thought that is established in the critical doctrine does not contradict this result in any way, but rather it confirms it and brings it to its sharpest expression. It is indeed at first glance strange and paradoxical that the most diverse epistemological standpoints, that radical empiricism and positivism as well as critical idealism have all appealed to the theory of relativity in support of their fundamental views. But this is satisfactorily explained by the facts that empiricism and idealism meet in certain presuppositions with regard to the doctrine of empirical space and of empirical time, and that the theory of relativity sets up just such a doctrine. Both here grant to experience the decisive rôle, and both teach that every exact measurement presupposes universal empirical *laws*.²⁵ But the question becomes all the more urgent as to how we reach these laws, on which rests the possibility of all empirical measurement, and what sort of validity, of logical "dignity" we grant to them. Strict positivism has only one

²⁴ On the "relativization" of the difference of space and time, *cf.* also below, VII.

²⁵ (8), p. 191ff.; *cf.* Sellien (81), p. 14ff.

answer to this question: for it all knowledge of laws, like all knowledge of objects, is grounded in the simple elements of sensation and can never go beyond their realm. The knowledge of laws possesses accordingly in principle the same purely passive character that belongs to our knowledge of any particular sensuous qualities. Laws are treated like things whose properties one can read off by immediate perception. Mach attempts, quite consistently with his standpoint, to extend this manner of consideration to pure mathematics also and the deduction of its fundamental relations. The way in which we gain the differential quotient of a certain function, as he explains, is not distinguished in principle from the way in which we establish any sort of properties or changes of physical things. As in the one case we subject the thing, so in the other case we subject the function to certain operations and simply observe how it "reacts" to them. The reaction of the function $y=x^m$ to the operation of differentiation out of which the equation $\frac{dy}{dx}=mx^{m-1}$ results "is a distinguishing mark of x^m just as much as the blue-green color in the solution of copper in sulphuric acid." (49, p.75.) Here we find clearly before us the sharp line of distinction between critical idealism and positivism of Mach's type. That the equations governing larger or smaller fields are to be regarded as what is truly permanent and substantial, since they make possible the gaining of a stable picture of the world,²⁶ that they thus constitute the kernel of physical objectivity: this is the fundamental view in which the two theories combine. The question concerns only the manner of establishing, only the exact grounding, of these equations. Idealism urges against the standpoint of "pure experience" as the standpoint of mere sensation, that all equations are results of measurement; all measurement, however, presupposes certain theoretical principles and in

²⁶ See Mach (49), p. 429.

the latter certain universal functions of connection, of shaping and coördination. We never measure mere sensations, and we never measure with mere sensations, but in general to gain any sort of relations of measurement we must transcend the "given" of perception and replace it by a conceptual symbol, which possesses no copy in what is immediately sensed. If there is anything that can serve as a typical example of this state of affairs, it is the development of modern physics in the theory of relativity. It is verified again that every physical theory, to gain conceptual expression and understanding of the facts of experience, must free itself from the form in which at first these facts are immediately given to perception.²⁷ That the theory of relativity is founded on experience and observation is, of course, beyond question. But, on the other hand, its essential achievement consists in the new interpretation that it gives to the observed facts, in the conceptual interpretation by which it is progressively led to subject the most important intellectual instruments of classical mechanics and the older physics to a critical revision. It has been pointed out with justice that it has been precisely the oldest empirical fact of mechanics, the equality of inert and heavy masses, which, in the new interpretation it has received from Einstein, has become the fulcrum of the general theory of relativity. (24a.) The way in which the principle of equivalence and with it the foundations of the new theory of gravitation have been deduced from this fact can serve as a logical example of the meaning of the pure "thought-experiment" in physics. We conceive ourselves in the position of an observer, who, experimenting in a closed box, establishes the fact that all bodies left to themselves move, always with constant acceleration, toward the floor of the box. This fact can be represented con-

²⁷ Cf. Duhem (15, p. 322): "*Les faits d'expérience, pris dans leur brutalité native, ne sauraient servir au raisonnement mathématique; pour alimenter ce raisonnement, ils doivent être transformés et mis sous forme symbolique.*"

ceptually by the observer in a double manner: in the first place, by the assumption that he is in a temporarily constant field of gravity in which the box is hung up motionless, or, in the second place, by the assumption that the box moves upward with a constant acceleration whereby the fall of bodies in it would represent a movement of inertia. The two: the inertial movement and the effect of gravitation, are thus in truth a single phenomenon seen and judged from different sides. It follows that the fundamental law that we establish for the movement of bodies must be such that it includes equally the phenomena of inertia and those of gravitation. As is seen, we have here no empirical proposition abstracted from particular observations, but a rule for our construction of physical concepts: a demand that we make, not directly of experience, but rather of our manner of intellectually representing it. "Thought-experiments" of such force and fruitfulness cannot be explained and justified by the purely empiristic theory of physical knowledge. It is not in contradiction with this that Einstein refers gratefully to the decisive stimulus, which he received from Mach (20); for a sharp distinction must be made between what Mach has accomplished as a physicist in his criticism of Newton's fundamental concepts, and the general philosophical consequences he has drawn from this achievement. Mach himself has, as is known, granted wide scope to the pure "thought-experiment" in his own logic of physics; but, more closely considered, he has thereby already left the ground of a purely sensualistic founding of the fundamental concepts of physics.²⁸ That there is no necessary connection between the theory of relativity and Mach's philosophy may be concluded from the fact, among other things, that it is precisely one of the first advocates of this theory, Max Planck, who among all modern physicists has most sharply criticized and fought

²⁸ See Mach (50, p. 180ff.); *cf.* (8), p. 316ff. and (39), p. 86f.

against the presuppositions of this philosophy. (69.) Even if one takes the theory of relativity as an achievement and outcome of purely empirical thought, it is thereby a proof and confirmation of the constructive force immanent in this thought by which the system of physical knowledge is distinguished from a mere "rhapsody of perceptions."

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